# Lösungen zu Kapitel 1.3, Gleichungen

## Kapitel 1.3.1, Quadratische Gleichungen

a)    1)    $L = \{1;2\}$             2)    $L = \{2\}$

      3)    für $a = 0$                 $\Rightarrow$         $L = (-\infty; \infty)$

              für $a \neq 0, a \neq b, a \neq -b$    $\Rightarrow$         $L = \left\{0; \dfrac{2b}{a+b}\right\}$

      4)    $L = \left\{0; \dfrac{2ab}{a+b}\right\}$         5)    $L = \left\{\dfrac{5}{2}; 5\right\}$

      6)    $L = \{-10; 4\}$           7)    $L = \emptyset$

      8)    $L = \{-3; 5\}$            9)    $L = \{-4; 13\}$

b)    1)    $L = \{-4; 3\}$           2)    $L = \{-1; 1\}$

      3)    $L = \{-3; -2; 2; 3\}$       4)    $L = \left\{-\dfrac{3}{2}; \dfrac{3}{2}\right\}$

      5)    $L = \{-4; 4\}$           6)    $L = \left\{0; \dfrac{2ab}{a+b}\right\}$

c)    1)    $D = R \backslash \left\{\dfrac{3}{2}\right\};\ L = \{\emptyset\}$      2)    $D = R \backslash \{1; 2; 3; 4\};\ L = \left\{\dfrac{5}{2}; 5\right\}$

      3)    $D = R \backslash \{-2; 3\};\ L = \{-3\}$     4)    $D = R;\ L = \{1; 2\}$

      5)    $D = R - \{-2; 1\};\ L = \{2\}$     6)    $D = R \backslash \left\{\dfrac{11}{2}; \dfrac{1}{3}\right\};\ L = \{-10; 4\}$

      7)    $D = R \backslash \{2; 3\};\ L = \{\emptyset\}$      8)    $D = R \backslash \left\{\dfrac{3}{2}\right\};\ L = \{-3; 5\}$

      9)    $D = R \backslash \{-5; -3\};\ L = \{-4; 13\}$    10)   $D = R;\ L = \{-3; 0; 1\}$

      11)   $D = R;\ L = \{-2; 2\}$         12)   $D = R;\ L = \left\{\dfrac{1}{2} \pm \dfrac{1}{2}\sqrt{1 - \dfrac{4}{a}}\right\}$

## Kapitel 1.3.2, Bi-quadratische Gleichungen

1) $L = \{-3; -1; 1; 3\}$        2) $L = \{-2; 2\}$

3)    1. Fall: $\quad a = 0 \qquad \Rightarrow \qquad$ keine Lösung

     2. Fall: $\quad a \neq 0 \qquad \Rightarrow \qquad x_1 = \dfrac{1}{2} + \dfrac{1}{2}\sqrt{1 - \dfrac{4}{a}} \qquad x_2 = \dfrac{1}{2} - \dfrac{1}{2}\sqrt{1 - \dfrac{4}{a}}$

## Kapitel 1.3.4, Wurzelgleichungen

a)    1) $\quad L = \dfrac{6n}{5b^4 m^5}$        2) $\quad L = uv^3 \cdot \sqrt[12]{u^9 v^8}$

     3) $\quad L = 2 \cdot (a - b)$        4) $\quad L = \dfrac{10}{3}\left(\dfrac{bc}{xy^2 z^2}\right)^n$

     5) $\quad L = \dfrac{4\sqrt[4]{ab}\left(\sqrt{a} + \sqrt{2b}\right)}{a^2 - 4b^2}$

b)

1) $\quad 4 \cdot \left(1 - x^2\right)^2 (x+5)\left(x^2 + 2x - 1\right)^2 = \left(1 - x^2\right)^2\left(x^2 + 2x - 1\right)^2 + (x+5)\left(x^2 + 2x - 1\right)^2 - (x - 1)$

     2) $\quad L = \left\{\dfrac{5}{2}\right\}$        3) $\quad L = \varnothing$

     4) $\quad L = \{1\}$        5) $\quad L = \{3\}$

     6) $\quad L = \{15\}$        7) $\quad L = \{12\}$

     8) $\quad L = \{7\}$        9) $\quad L = \{2\}$

     10) $\quad L = \{1\}$        11) $\quad L = \{34\}$

     12) $\quad L = \varnothing$        13) $\quad L = \{9\}$

     14) $\quad L = \{1\}$        15) $\quad L = \left\{\dfrac{1}{4}; \dfrac{1}{2}\right\}$

     16) $\quad L = \{6\}$        17) $\quad L = \{2\}$

     18) $\quad L = \{5\}$        19) $\quad L = \left\{-1; \dfrac{9}{16}\right\}$

c)  1)  $L = \{1\}$            2)  $L = \{5\}$

   3)  $L = \left\{\dfrac{ab}{a+b}\right\}$      4)  $L = \{11\}$

   5)  $L = \{3;4\}$         6)  $L = \{3;4\}$

   7)  $L = \left\{\dfrac{1}{4};\dfrac{1}{2}\right\}$

d)  1)  $L = \sqrt[8]{2^7}$        2)  $L = \dfrac{\sqrt{u}}{\sqrt[3]{v^2}}$

   3)  $L = \sqrt[6]{2}$          4)  $L = \dfrac{1}{a^2 bx}$

   5)  $L = a$             6)  $L = \sqrt[6]{mn^3}$

   7)  $L = \{6\}$          8)  $L = \{2\}$

   9)  $L = \{-2;1\}$        10)  $L = \{3\}$

## Kapitel 1.3.5, Exponentialgleichungen

a)  1)  $L = \left\{1;\dfrac{7}{4}\right\}$        2)  $L = \{1\}$

   3)  $L = \{0{,}435\}$       4)  $L = \{1{,}136\}$

   5)  $L = \left\{\dfrac{n\lg a - \lg 2}{\lg a + \lg b}\right\}$    6)  $L = \left\{\dfrac{p\lg a - q\lg b}{m\lg a - n\lg b}; m\cdot\lg a - n\cdot\lg b \neq 0\right\}$

   7)  $L = \{-2{,}738\}$      8)  $L = \{\varnothing\}$

b)  1)  $L = \{-7;1\}$        2)  $L = \left\{-\dfrac{2}{5}\right\}$

   3)  $L = \{0{,}414\}$       4)  $L = \{-0{,}414\}$

   5)  $L = \{-0{,}342\}$      6)  $L = \left\{\dfrac{s\lg n + t\lg m}{r\lg n + 2q\lg m}; r\lg n + 2q\lg m \neq 0\right\}$

   7)  $L = \{-0{,}414\}$      8)  $L = \{16\}$

c) 1) $L = \left\{\dfrac{3}{2}\right\}$  2) $L = \{35\}$

3) $L = \{-9; 4\}$  4) $L = \left\{1; \dfrac{7}{4}\right\}$

5) $L = \left\{\dfrac{\lg 2}{\lg 2 - \lg 3}\right\}$  6) $L = \{1\}$

7) $L = \left\{\dfrac{\lg \dfrac{\lg 3}{\lg 2}}{\lg 3 - \lg 2}\right\}$  8) $L = \left\{\dfrac{\lg 22 - \lg 25}{\lg 15 - \lg 17}\right\}$

9) $L = \{-1; 1\}$  10) $L = \{0\}$

11) $L = \{11\}$  12) $L = \{3,3535\}$

13) $L = \left\{1 + \ln\left(1 + \sqrt{1+e}\right)\right\}$  14) $L = \left\{-\dfrac{1}{2}\right\}$

15) $L = \left\{2\dfrac{1}{4}\right\}$  16) $L = \{-4\}$

17) $L = \{3\}$  18) $L = \{-3; 2\}$

19) $L = \{11\}$  20) $L = \{-1,7\}$

## Kapitel 1.3.6, Logarithmische Gleichungen

a) 1) $L = \{2\}$  2) $L = \left\{1 + e^2\right\}$

3) $L = \left\{ae^b\right\}$  4) $L = \{998\}$

5) $L = \left\{\dfrac{1}{998}\right\}$  6) $x^4 + 9x^2 - 5x + 28 = 0$

7) $L = \{8\}$  8) $L = \{1; 5\}$

9) $L = \left\{\pm\dfrac{1}{2}\sqrt{2 - \sqrt{e}}\right\}$  10) $L = \left\{\dfrac{1}{2} \pm \sqrt{4,25}\right\}$

11) $L = \left\{\dfrac{32}{a}\right\}$  12) $L = \{27\}$

13) $L = \{32\}$

14) $L = \{3\}$

15) $L = \{10^{-4}; 10\}$

16) $L = \{-1; +1\}$

17) $L = \{10; 10^4\}$

18) $L = \{4\}$

19) $L = \{100\}$

20) $L = \{8\}$

21) $L = \{-8; 8\}$

22) $L = \{u^2 \cdot v^3\}$

23) $L = \left\{ \dfrac{a}{b} \right\}$

24) $A = \dfrac{F}{B^{C \cdot \lg D + E}}; \quad B = 10^{\frac{\lg F - \lg A}{C \cdot \lg D + E}}$

24) $C = \dfrac{\lg F - \lg A - E \cdot \lg B}{\lg B \cdot \lg D}; \quad D = 10^{\frac{\lg F - \lg A - E \cdot \lg B}{C \cdot \lg B}}; \quad E = \dfrac{\lg F - \lg A - C \cdot \lg D \cdot \lg B}{\lg B}$

25) $L = \{3\}$

26) $L = \{-2; 2\}$

27) $L = \left\{ e^3; e^{-\frac{4}{3}} \right\}$

28) $L = \{5\}$

29) $L = \{49\}$

30) $L = \{125\}$

31) $L = \left\{ e^{e^2} \right\}$

32) $L = \{-100; 100\}$

33) $L = \{5000\}$

34) $L = \{8\}$

35) $L = \{2(e^3 + 1)\}$

36) $L = \{1\}$

37) $L = \{100 \cdot \sqrt{10}\}$

38) $L = \{\sqrt[5]{5}; 5\}$

39) $L = \{100\}$

40) $L = \{6^{-1}; 6\}$

41) $L = \{10; 10^4\}$

42) $L = \{-0{,}6; 2\}$

43) $L = \{1\}$

44) $L = \{0\}$

45) $L = \{2\}$

46) $L = \left\{ \dfrac{3}{2} \right\}$

47) $L = \left\{ \dfrac{3}{2} \right\}$

b)  1)  $L = 3 + \dfrac{1}{3}\ln(a+b)$       2)  $L = \dfrac{1}{3}\left[2 + \lg(a+b) - 2\lg c\right]$

    3)  $L = \dfrac{1}{5}\ln a - \dfrac{7}{4}\ln c + \dfrac{7}{3}$       4)  $L = \dfrac{1}{5}\lg a + \dfrac{1}{2}\lg(a+b)$

    5)  $L = \dfrac{1}{6}\lg a^3 b$       6)  $L = \lg \dfrac{\sqrt{c}(a+b)}{a^4(a-b)}$

    7)  $L = \lg\left[2(a+b)^2\right]$       8)  $L = 4 + \dfrac{1}{3}\left[\ln(a-b) - \ln(a+b)\right]$

c)  1)  $L = \left\{\dfrac{13}{3}\right\}$       2)  $L = \left\{\dfrac{9}{2}\right\}$

    3)  $L = \left\{\dfrac{31}{6}\right\}$       4)  $L = \{0\}$

    5)  $L = \left\{\dfrac{7}{3}\right\}$       6)  $L = \{-5;2\}$

    7)  $L = \{13\}$       8)  $L = \{998\}$

    9)  $L = \left\{\dfrac{1}{998}\right\}$       10)  $L = \left\{ae^b\right\}$

    11)  $L = \left\{\dfrac{32}{a}\right\}$       12)  $L = \{27\}$

    13)  $L = \{32\}$       14)  $L = \{3\}$

    15)  $L = \left\{10^{-4};10\right\}$       16)  $L = \left\{10;10^4\right\}$

    17)  $L = \{-1;1\}$       18)  $L = \left\{-\dfrac{7}{8}\right\}$

    19)  $L = \{8\}$       20)  $L = \{4\}$

**Kapitel 1.3.7, Trigonometrische (Goniometrische) Gleichungen**

a)   1)   $x_1 = 2k\boldsymbol{p}, \quad k = \pm 0,1,2,....$   2)   $x_1 = \boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_2 = \dfrac{\boldsymbol{p}}{2} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   $x_{2,3} = \pm\dfrac{\boldsymbol{p}}{3} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

3)   $x_{1,2} = \pm\dfrac{\boldsymbol{p}}{3} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   4)   $x_1 = \dfrac{\boldsymbol{p}}{4} + k\boldsymbol{p}, \quad \left(45° + k\boldsymbol{p} \quad k = +0,1,2...\right)$

$x_2 = 0{,}4194\boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   $x_2 = 0{,}8976\boldsymbol{p} + k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

5)   $x_1 = 0{,}3524\boldsymbol{p} + k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   6)   $x_1 = 0{,}7048\boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_2 = 0{,}0675\boldsymbol{p} + k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   $x_2 = 1{,}2961\boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_3 = 0{,}6825\boldsymbol{p} + k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

7)   $x_{1,2} = \pm\dfrac{\boldsymbol{p}}{4} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   8)   $L = \{\varnothing\}$

$x_3 = \dfrac{3}{4}\boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_4 = \dfrac{5}{4}\boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

9)   $x = 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   10)   $x_1 = \dfrac{\boldsymbol{p}}{2} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_2 = \boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

11)   $L = \{\varnothing\}$   12)   $x_{1,2} = \pm 0{,}1795\boldsymbol{p} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_3 = k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

13)   $x = \dfrac{\boldsymbol{p}}{2} + k\boldsymbol{p}, \quad k = \pm 0,1,2,...$   14)   $x_1 = -\dfrac{\boldsymbol{p}}{4} + k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_2 = 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

$x_3 = -\dfrac{\boldsymbol{p}}{2} + 2k\boldsymbol{p}, \quad k = \pm 0,1,2,...$

b)  1)  $L = \{30° + k \cdot 360°; 150° + k \cdot 360°; k \in G\}$

2)  $L = \{0°; 32,3°; 147,7°; 180°; 212,3°; 327,7°\}$

3)  $L = \left\{ \dfrac{\boldsymbol{p}}{6}; \dfrac{\boldsymbol{p}}{3}; \dfrac{7}{6}\boldsymbol{p}; \dfrac{4}{3}\boldsymbol{p} \right\}$

4)  $L = \{38,7°; 321,3°\}$

5)  $L = \{0°; 233,1°\}$

6)  $L = \left\{ \begin{array}{l} 90° + k \cdot 180°; 228°35' + k \cdot 360°; \\ 311°25' + k \cdot 360°; k \in G \end{array} \right\}$

7)  $L = \left\{ k\boldsymbol{p}; \dfrac{\boldsymbol{p}}{4} + k\boldsymbol{p}; k \right\}$

c)  1)  $L = \{2\cos x\}$

2)  $L = \{\cos\boldsymbol{a}\}$

3)  $L = \dfrac{\cos\boldsymbol{a} + \sin\boldsymbol{a}}{\cos\boldsymbol{a} - \sin\boldsymbol{a}}$

4)  $L = \{0\}$

5)  $L = \{\cot\boldsymbol{a}\}$

d)  1)  $L = \{240° + k \cdot 360°; 300° + k \cdot 360°; k\}$

2)  $L = \left\{ \dfrac{\boldsymbol{p}}{4} + k\boldsymbol{p}; k\boldsymbol{p} \right\}$

3)  $L = \{60°; 540°\}$

4)  $L = \{75,5°; 284,5°\}$

5)  $L = \{104,5°; 255,5°\}$

6)  $L = \{0°; 101°; 180°; 259°\}$

7)  $L = \{60°; 120°; 240°; 300°\}$