Lösungen zu Kapitel 4.2, Technik des Differenzierens

Kapitel 4.2.2, Differentiationsregeln

a) 1) $y' = 30x^2$ 2) $y' = 28x^6 + 3\cos x$

3) $y' = \left(12x^2 - 3\right)\left(2e^x - \sin x\right) + \left(2e^x - \cos x\right)\left(4x^3 - 3x\right)$

4) $y' = \dfrac{\left(3x^2 - 4\right)\left(2x^2 - 4x + 1\right) - (4x - 4)\left(x^3 - 4x + 5\right)}{\left(2x^2 - 4x + 1\right)^2}$

5) $y' = \dfrac{x\cos x - 2\sin x}{x^3}$ 6) $y' = x \cdot (2\cos x - x\sin x)$

7) $y' = 15\cos(5x)$ 8) $y' = 24(3x - 4)^7$

9) $y' = (8x - 3) \cdot e^{4x^2 - 3x + 2}$ 10) $y' = \dfrac{\sin\left[2\left(wx + j\right)\right] \cdot e^{\sin^2(wx + j)}}{2\sqrt{e^{\sin^2(wx - j)}}}$

11) $y' = \dfrac{20x}{1 + x^2}$ 12) $y' = -\dfrac{2a^2 x}{\sqrt{\left(a^2 - x^2\right)\left(a^2 + x^2\right)^3}}$

b) 1) $y' = 2\cos(2x)$ 2) $y' = (2x + 3) \cdot \cos\left(x^2 + 3x\right)$

3) $y' = -4x\left(x^2 - 2\right) \cdot \sin\left(\left(x^2 - 2\right)^2\right)$ 4) $y' = -\sin(2x)$

5) $y' = -(2x + 3) \cdot \sin\left(2x^2 + 6x\right)$ 6) $y' = 10e^x \cdot \sin^3\left(e^x\right) \cdot \sin\left(2e^x\right)$

7) $y' = 6 \cdot \left(e^x + 4x\right) \cdot \cos^2\left(e^x + 2x^2\right) \cdot \sin\left(2e^x + 4x^2\right)$

8) $y' = \dfrac{e^{\sin x}(x\cos x - 2)}{x^3}$ 9) $y' = \dfrac{3x - (1 + 3x) \cdot \ln(1 + 3x)}{x^2(1 + 3x)}$

10) $y' = 2\sqrt{x^2 - 25}$ 11) $y' = -\ln 10 \cdot \sin x \cdot 10^{\cos x}$

12) $y' = -\ln 10 \cdot 10^x \cdot \sin\left(10^x\right)$ 13) $y' = \cos 10 \cdot x^{\cos 10 - 1}$

14) $y' = \dfrac{\ln 2}{x} \cdot 2^{\ln x}$ 15) $y' = \ln 2$

16) $y' = \dfrac{2}{x} \cdot \ln x$ 17) $y' = x^{\cos x} \cdot \left(\dfrac{\cos x}{x} - \sin x \cdot \ln x\right)$

c) 1) $y' = a^x x^{a-1}(x \cdot \ln a + a)$ 2) $y' = e^{x^2} \cdot 2x$

3) $y' = \dfrac{x^{a-1}(a - x \cdot \ln a)}{a^x}$ 4) $y' = \dfrac{a^x(x \cdot \ln a - a)}{x^{a+1}}$

5) $y' = 2e^{2x}$ 6) $y' = 5e^{5x}$

7) $y' = -20b^2x^4(a^3 - b^2x^5)^3$ 8) $y' = 1$

9) $y' = \dfrac{2x}{x^2 - 2}$ 10) $y' = \dfrac{\cos^2 x}{2\sqrt{\tan x}}$

11) $y' = \dfrac{2x \cdot \lg e}{x^2 + 1}$ 12) $y' = \dfrac{e^{2x}(1-x)+1}{(1+e^{2x})^{\frac{3}{2}}}$

13) $y' = \dfrac{-4x}{\cos^2(p - 2x^2)}$ 14) $y' = \dfrac{2x}{1 - x^4}$

15) $y' = e^x \cdot \cos(e^x)$ 16) $y' = e^{\sin x} \cdot \cos x$

17) $y' = -\dfrac{1}{\cos x}$ 18) $y' = \dfrac{x}{1 + x^2}$

19) $y' = e^{\frac{x-1}{x+1}} \cdot \dfrac{2}{(x+1)^2}$ 20) $y' = \dfrac{e^{2x+1}}{\sqrt{e^{2x+1}} \cdot \cos^2\sqrt{e^{2x+1}}}$

21) $y' = 2a^{2x} \cdot \ln a \cdot \cot(a^{2x} + 1)$ 22) $y' = (3e)^x$

23) $y' = \dfrac{a^{\sqrt{x}} \cdot \ln a}{2\sqrt{x}}$ 24) $y' = a^{-3x}\left(\dfrac{1}{\sqrt{2x-5}} - 3\sqrt{2x-5} \cdot \ln a\right)$

25) $y' = 2^x \cdot \ln 2 + 2x + ax^{n-1}$ 26) $y' = b(a^b)^x \cdot \ln a$

27) $y' = abx^{ab-1}$ 28) $y' = -\tan x$

29) $y' = -\dfrac{xe^{\sqrt{1-x^2}}}{\sqrt{1-x^2}}$ 30) $y' = \dfrac{1}{\cos x}$

31) $y' = 2 \cdot \dfrac{1 + \tan^2 x}{1 - \tan^2 x}$ 32) $y' = \dfrac{3 \cdot (2x^2 + \sqrt{x})}{4\sqrt{x^3 + \sqrt{x^3}}}$

33) $y' = \dfrac{e^{2x}}{\sqrt{e^{2x+1}} \cdot \cos^2 \sqrt{e^{2x+1}}}$

34) $y' = \dfrac{x \cdot \cos x - \sin x}{2x \cdot \sin x \cdot \sqrt{\ln \dfrac{\sin x}{x}}}$

35) $y' = -\dfrac{2e^{\sqrt[3]{\sin(-2x)}} \cdot \cos(-2x)}{3 \cdot \sqrt[3]{\sin^2(-2x)}}$

36) $y' = \dfrac{12x \cdot \sin(2x^2 - 3)}{\cos^4(2x^2 - 3)}$

37) $y' = \dfrac{2x}{(x^2 - 1) \cdot \ln 3}$

38) $y' = e^{\sqrt{x+1}} \cdot \dfrac{1}{2\sqrt{x+1}}$

39) $y' = \ln 2 \cdot 2^x \cdot \cos 2^x$

40) $y' = \cos(\sin x) \cdot \cos x$

41) $y' = -12 \cos^2 4x \cdot \sin 4x$

d) 1) $y' = \dfrac{1}{3 \cdot \sqrt[3]{x^2}}$

2) $y' = \dfrac{4}{5 \cdot \sqrt[5]{x}}$

3) $y' = \dfrac{-1}{2 \cdot \sqrt{x^3}}$

4) $y' = \dfrac{5}{3} \cdot \sqrt[3]{x^2}$

5) $y' = 4x + 3$

6) $y' = 6x^2 - 5$

7) $y' = 6x^2 + 2x - 6$

8) $y' = 6x^2 - 42x + 60$

9) $y' = \dfrac{3}{2 \cdot \sqrt{x}} - \dfrac{1}{2 \cdot \sqrt{x^3}}$

10) $y' = 2x - 3 + \dfrac{1}{x^2}$

11) $y' = \dfrac{-1}{3 \cdot \sqrt[3]{x^4}} + \dfrac{4}{3 \cdot \sqrt[3]{x^5}}$

12) $y' = -\dfrac{a}{x^2}$

13) $y' = \dfrac{-2}{1 + t^2}$

14) $y' = \dfrac{-2x^2 + 2x - 3}{(x^2 + x - 2)^2}$

15) $y' = 4x + 1$

16) $y' = \sin x + x \cdot \cos x$

17) $y' = \tan x + \dfrac{x - 1}{\cos^2 x}$

18) $y' = \dfrac{1}{\sqrt{x} \cdot (\sqrt{x} + 1)^2}$

19) $y' = \dfrac{1 + 2x + 3x^2 - 2x^3 - x^4}{(x^3 + 1)^2}$

20) $y' = \dfrac{4x^3 - 1}{\sqrt{2}}$

21) $y' = \dfrac{1}{\sin^2 x \cdot \cos^2 x}$

22) $y' = 2 \cdot \dfrac{1 - \cos x - x \cdot \sin x}{(1 - \cos x)^2}$

23) $y' = \dfrac{2}{1+\sin 2x}$

24) $y' = \cot x - \dfrac{x}{\sin^2 x}$

25) $y' = \cos x - \sin x$

26) $y' = x \cdot (2\cos x - x \cdot \sin x)$

27) $y' = \dfrac{4x - \sin 2x}{4x \cdot \sqrt{x} \cdot \cos^2 x}$

28) $y' = \dfrac{\sin x - 1}{\cos^2 x}$

29) $y' = \dfrac{\cos x - 2x \cdot \sin x}{2 \cdot \sqrt{x}}$

30) $y' = \dfrac{1}{2} + \dfrac{2}{x^2}$

31) $y' = \dfrac{\cos x}{2 \cdot \sqrt{\sin x}}$

32) $y' = \dfrac{-2x}{\sqrt{1-x^2}}$

33) $y' = \dfrac{-\cos\dfrac{1}{x}}{x^2}$

34) $y' = \dfrac{\cos\sqrt{x}}{2 \cdot \sqrt{x}}$

35) $y' = 10 \cdot (2x-1)^4$

36) $y' = \pm\left(\sqrt{1+\sin 2x} + \sqrt{1-\sin 2x}\right)$

37) $y' = \dfrac{1-\cos 2x}{\sqrt{2x - \sin 2x}}$

38) $y' = \dfrac{1 + 2x \cdot \cos^2 x}{2 \cdot \cos^2 x \cdot \sqrt{\tan x + x^2}}$

39) $y' = \dfrac{-2}{(1+x)^2} \cdot \sin 2 \cdot \dfrac{1-x}{1+x}$

40) $y' = \dfrac{1-x^2}{2x^2 \cdot \sin^2\left(x + \dfrac{1}{x}\right) \cdot \sqrt{\cot\left(x + \dfrac{1}{x}\right)}}$

41) $y' = \dfrac{\sin 2 \cdot \dfrac{1-\sqrt{x}}{1+\sqrt{x}}}{\sqrt{x} \cdot (1+\sqrt{x})^2}$

42) $y' = \dfrac{-3p}{2 \cdot \sin^4(p \cdot x)}$

43) $y' = 4 \cdot \dfrac{\cos x + \sin x}{(\cos x - \sin x)^3}$

44) $y' = \dfrac{\cos(3x-2)}{\sqrt[3]{\sin^2(3x-2)}}$

45) $y' = \dfrac{7 \cdot \sin^3 x}{\cos^2 x}$

46) $y' = \dfrac{(a-b) \cdot \sin 2x}{2 \cdot \sqrt{a \sin^2 x + b \cos^2 x}}$

47) $y' = 6x \cdot \sin^2(x^2) \cdot \cos(x^2)$

48) $y' = \dfrac{-1}{2 \cdot (1+\sqrt{x}) \cdot \sqrt{x(1-x)}}$

49) $y' = \dfrac{2}{3} \cdot \sqrt[3]{\dfrac{1+\sin 2t}{\cos^4 2t}}$

50) $y' = \dfrac{1}{2x \cdot \sqrt{x}} \cdot \sin 2 \cdot \sqrt{\dfrac{1}{x}}$

51) $y' = \dfrac{x \cdot (x^3+2)}{\left(\sqrt[3]{x^3+1}\right)^4}$

52) $y' = \dfrac{-1}{(1+x^2)^{\frac{3}{2}}}$

Kapitel 4.2.3, Die Differentiation impliziter Funktionen

1) $y' = \dfrac{4-3x^2}{y^2+1}$

2) $y' = \dfrac{5x^4}{2y}$

3) $y' = -\dfrac{a}{x^2}$

4) $y' = \dfrac{c}{x^2 \cdot \sin y}$

5) $y' = \dfrac{3x^2 - 2xe^y}{2y + x^2 e^y}$

6) $y' = \dfrac{y - \cos x}{\cos y - x}$

7) $y' = \dfrac{3 \cdot (y-x)^2}{3 \cdot (y-x)^2 - \sin 2y}$

8) $y' = \dfrac{6x - 15x^2 \cdot (x^3 + y^2)^4}{10y \cdot (x^3 + y^2)^4}$

9) $y' = \dfrac{\cos^2 y}{x \cdot \sin 2y - 3y^2}$

10) $y' = \dfrac{y}{2} \cdot \left[\dfrac{\ln(\sin x)}{x} + \ln x \cdot \cot x \right]$

11) $y' = \dfrac{y^2 - xy \cdot \ln x}{x^2 - xy \cdot \ln x}$

Kapitel 4.2.4, Differentiation durch Logarithmieren

1) $y' = 4xe^{2x^2}$

2) $y' = x^x \cdot (\ln x + 1)$

3) $y' = (\sin 2x)^{\sqrt{\cos 3x}} \cdot \sqrt{\cos 3x} \cdot \left[2 \cdot \cot 2x - \dfrac{3}{2} \cdot \tan 3x \cdot \ln(\sin 2x) \right]$

4) $y' = (\ln x)^x \cdot \left[\ln(\ln x) + \dfrac{1}{\ln x} \right]$

5) $y' = x^{\sqrt{x} - \frac{1}{2}} \cdot (\ln x + 2)$

6) $y' = 2 \cdot \sqrt[x]{(x+1)^2} \cdot \left[\dfrac{1}{x \cdot (x+1)} - \dfrac{\ln(x+1)}{x^2} \right]$

7) $y' = x^{\sin x} \cdot \left(\cos x \cdot \ln x + \dfrac{\sin x}{x} \right)$

8) $y' = \sin x^{\cos x} \cdot \left[\dfrac{\cos^2 x}{\sin x} - \sin x \cdot \ln(\sin x) \right]$

9) $y' = \left(\dfrac{x}{x+1} \right)^x \cdot \left(\ln \dfrac{x}{x+1} + \dfrac{1}{x+1} \right)$

10) $y' = x^{x^2+1} \cdot (\ln x^2 + 1)$

11) $y' = x^{\frac{1}{x}-2} \cdot (1 - \ln x)$

12) $y' = x^{x^x + x} \cdot \left(\ln^2 x + \ln x + \dfrac{1}{x} \right)$

13) $y' = \frac{1}{3} x \cdot \left(\sqrt[3]{x}\right)^{x^2} \cdot (2 \cdot \ln x + 1)$

14) $y' = \frac{y}{x} \cdot \left(\frac{1}{x} - \frac{\ln x}{x} - 1\right)$

15) $y' = \frac{x^{\sqrt{x}}}{\sqrt{x}} \cdot \left(1 + \frac{1}{2} \cdot \ln x\right)$

16) $y' = \frac{\sqrt{x}^{x}}{2} \cdot (1 + \ln x)$

17) $y' = \frac{\sqrt[x]{x}}{x^2} \cdot (1 - \ln x)$

18) $y' = (\cos x)^{x} \cdot \left[\ln(\cos x) - x \cdot \tan x\right]$

19) $y' = \left(1 + \frac{1}{x}\right)^{x} \cdot \left[\ln\left(1 + \frac{1}{x}\right) - \frac{1}{1+x}\right]$