

Lösungen zu Kapitel 4.4, Höhere Ableitungen von Funktionen

- a) 1) $y' = 2x \cdot (\sin 2x + x \cdot \cos 2x)$ $y'' = 2 \cdot (1 - 2x^2) \cdot \sin 2x + 8x \cdot \cos 2x$
- 2) $y' = 4 \cdot \frac{x^2 - 3x}{(x^2 + 2x - 3)^2}$ $y'' = 4 \cdot \frac{-2x^3 + 9x^2 + 9}{(x^2 + 2x - 3)^3}$
- 3) $y' = \frac{x+1}{(1+2x)^{\frac{3}{2}}}$ $y'' = -\frac{x+2}{\sqrt{(1+2x)^5}}$
- 4) $y' = e^x \cdot (\cos x + \sin x)$ $y'' = 2e^x \cdot \cos x$
- 5) $y' = n \cdot \ln a \cdot a^{nx}$ $y'' = (n \cdot \ln a)^2 \cdot a^{nx}$
- 6) $y' = x^4 \cdot (1 + 5 \cdot \ln x)$ $y'' = x^3 \cdot (9 + 20 \cdot \ln x)$
- 7) $y' = \frac{1}{(1-x)^2}$ $y'' = \frac{2}{(1-x)^3}$
- 8) $y' = \frac{2x^3 - 3x^2}{(x-1)^2}$ $y'' = \frac{2x^3 - 6x^2 + 6x}{(x-1)^3}$
- 9) $y' = -\frac{nx^{n-1}}{2\sqrt{1-x^n}}$ $y'' = -\frac{n}{4} \cdot \frac{2 \cdot (1-x^n)(n-1)x^{n-2} - nx^{2n-2}}{(\sqrt{1-x^n})^3}$
- 10) $y' = \frac{e^{\tan x}}{\cos^2 x}$ $y'' = \frac{e^{\tan x} \cdot (1 + \sin 2x)}{\cos^4 x}$
- 11) $y' = 2 \cdot \ln p \cdot p^{2x}$ $y'' = (2 \cdot \ln p)^2 \cdot p^{2x}$
- 12) $y' = -\frac{1}{k} \cdot (1-x)^{\frac{1}{k}-1}$ $y'' = \frac{k-1}{k^2} \cdot (1-x)^{\frac{1}{k}-2}$
- 13) $y' = \frac{a^2 x^2 + 2abx}{(ax+b)^2}$ $y'' = \frac{2ab^2}{(ax+b)^3}$

b) 1) $y' = \frac{2+8x-x^2}{(x^2+2)^2}$ $y'' = \frac{2x^3-24x^2-12x+16}{(x^2+2)^3}$

2) $y' = \frac{x^2-8x-2}{(x-4)^2}$ $y'' = \frac{36}{(x-4)^3}$

3) $y' = \frac{-2x^3+6x^2+7}{(x^3+3x+1)^2}$ $y'' = -\frac{2 \cdot (6-4x+9x^2+6x^3+8x^4-3x^5)}{(x^3+3x+1)^3}$

4) $y' = \frac{xe^x}{(x+1)^2}$ $y'' = \frac{(x^2+1)e^x}{(x+1)^3}$

5) $y' = \frac{2x \cdot \sin x - x^2 \cdot \cos x}{\sin^2 x}$
 $y'' = \frac{\sin^2 x \cdot (2+x^2) - 2x \cdot \cos x(2 \cdot \sin x - x \cdot \cos x)}{\sin^3 x}$

6) $y' = \frac{x \cdot (1 + \tan^2 x) - \tan x}{x^2}$ $y'' = \frac{2x \cdot (\tan^3 x - \tan^2 x + \tan x - 1) + 2 \tan x}{x^3}$

c) 1) $y' = 6x - \frac{1}{\sqrt{x}} + \frac{1}{3x^{\frac{4}{3}}}$ $y'' = \frac{1}{2\sqrt{x}^3} - \frac{4}{9x^{\frac{7}{3}}}$
 $y''' = -\frac{3}{4\sqrt{x}^5} + \frac{28}{27\sqrt[3]{x}^{10}}$

2) $y' = 36x^3 - 8x + 4$ $y'' = 108x^2 - 8$ $y''' = 216x$

3) $y' = 5\cos x - 3\sin x$ $y'' = -5\sin x - 3\cos x$ $y''' = -5\cos x + 3\sin x$

d) 1) $y' = -\cot t$ $y'' = \frac{1}{3\sin^4 t \cdot \cos t}$

2) $y' = 2t$ $y'' = -2$

3) $y' = e^{-2t}$ $y'' = -2e^{-3t}$

4) $y' = -\cot t$ $y'' = -\frac{1}{R\sin^3 t}$