Übungsaufgaben zu Kapitel 5.5, Das bestimmte Integral

Kapitel 5.5.2, Hauptsatz der Differential- und Integralrechnung

Lösen Sie folgende bestimmte Integrale

1) $\int_{-r}^{r} \sqrt{r^2 - x^2}\, dx$

2) $\int_{-a}^{a} \frac{1}{a^2 + x^2}\, dx$

3) $\int_{-a}^{a} a^x\, dx \quad \text{für} \quad a>0$

4) $\int_{0}^{\frac{a}{2}\cdot\sqrt{3}} \frac{1}{\sqrt{a^2 - x^2}}\, dx$

5) $\int_{0}^{4} x^2\cdot\sqrt{16 - x^2}\, dx$

6) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\cot(y)^2 - 5\right) dy$

7) $\int_{0}^{\frac{\pi}{4}} (\sin(\phi))^2\cdot\cos(\phi)\, d\phi$

8) $\int_{-2}^{1} \left(2\cdot x^3 - 3\cdot x^2 + 3\cdot x + 4\right) dx$

9) $\int_{0}^{a} e^{\frac{x}{a}}\, dx$

10) $\int_{0}^{1} (1 - x)^p\, dx$

11) $\int_{0}^{2} v^2\cdot\ln\left(20 - v^3\right) dv$

12) $\int_{0}^{\frac{\pi}{2}} \sin(x)\cdot e^{\cos(x)}\, dx$

13) $\int_{0,5}^{1} \frac{e^{\frac{1}{x} - 1}}{x^2}\, dx$

14) $\int_{\frac{\pi}{2}}^{-\pi} \sin(x)\, dx$

15) $\int_{-1}^{3} \left(7 \cdot x^2 + 26 - 2 \cdot x\right) dx$

16) $\int_{-1}^{3} \left(7 \cdot x^2 + 26 \cdot x - 2\right) dx$

17) $\int_{-4}^{-1} \left(3 \cdot x^2 - 4 \cdot x - 6\right) dx$

18) $\int_{1}^{8} \left(\sqrt[3]{x} + \sqrt[3]{x^2}\right) dx$

19) $\int_{0}^{2 \cdot \pi} (u + \sin(u))\, du$

20) $\int_{0}^{2 \cdot \pi} (1 + \sin(z))\, dz$

21) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos(2 \cdot v)}{\cos(v)^2 \cdot \sin(v)^2} dv$

22) $\int_{0}^{1} \frac{1}{\left(3 + 3 \cdot z^2\right)} dz$

23) $\int_{0}^{1} \frac{1}{\sqrt{4 - 4 \cdot u^2}} du$

24) $\int_{2}^{4} \left(e^x - 1\right) dx$

25) $\int_{1}^{3} \frac{1}{x + x^2} dx$

26) $\int_{0}^{\frac{\pi}{2}} \sin(x)^2 dx$

27) $\int_{0}^{\frac{\pi}{2}} \sin(x)^4 dx$

28) $\int_{0}^{\frac{\pi}{2}} \sin(x)^6 dx$

29) $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4 - x^2}} dx$

30) $\int_{0}^{a} \left(x^2 - a \cdot x\right) dx$

31) $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{1}{\cos(2\cdot x)^2}\,dx$

32) $\int_{0}^{\frac{\pi}{2}} x\cdot\cos(x)\,dx$

33) $\int_{1}^{4} \frac{1}{\left(1+\sqrt{x}\right)^2}\,dx$

34) $\int_{1}^{5} \frac{x}{\sqrt{4\cdot x+5}}\,dx$

35) $\int_{0}^{\frac{\pi}{2}} \cos(x)^2\,dx$

36) $\int_{0}^{\frac{\pi}{2}} \cos(x)^4\,dx$

37) $\int_{0}^{\frac{\pi}{2}} \cos(x)^6\,dx$

Kapitel 5.5.3, Das uneigentliche Integral

Bestimmen Sie das Konvergenzverhalten der folgenden uneigentlichen Integrale.

1) $\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

2) $\int_{0}^{\infty} \sin(x)\,dx$

3) $\int_{3}^{\infty} \frac{1}{9+u^2}\,du$

4) $\int_{0}^{\infty} e^{-z}\cdot\sin(\alpha\cdot z)\,dz$

5) $\int_{\frac{\pi}{4}}^{\frac{3}{4}\cdot\pi} (\tan(x))^2\,dx$

6) $\int_{1}^{\infty} \frac{1}{x^2}\,dx$

7) $\int_0^1 \frac{1}{x^2}\,dx$

8) $\int_1^\infty \frac{x+1}{\sqrt[3]{x^2}}\,dx$

9) $\int_0^\infty x\cdot e^{-2\cdot x}\,dx$

10) $\int_1^\infty \frac{1}{x^n}\,dx$

11) $\int_0^1 \frac{1}{x^n}\,dx$

12) $\int_\pi^\infty e^{-s}\cdot\sin(2\cdot s)\,ds$

13) $\int_{-1}^1 \frac{1}{\sqrt{1-u^2}}\,du$

14) $\int_3^5 \frac{1}{\sqrt{v^2-9}}\,dv$

15) $\int_0^1 \frac{1}{\sqrt{1-p}}\,dp$

16) $\int_2^3 \frac{6\cdot w^2}{\sqrt{w^2-8}}\,dw$

17) $\int_2^3 \frac{3\cdot z}{\sqrt{z^2-4}}\,dz$

18) $\int_0^{\frac{\pi}{4}} \frac{\sin(2\cdot x)}{1-\sin(2\cdot x)}\,dx$

19) $\int_0^2 \frac{6\cdot y}{3\cdot y^2-12}\,dy$

20) $\int_0^8 \frac{1}{\sqrt[3]{8-a}}\,da$

21) $\int_1^\infty e^{-\beta\cdot u}\,du$

22) $\int_0^1 \frac{1}{\sqrt[4]{1-w}}\,dw$

23) $\int_{-\infty}^\infty \frac{1}{\sqrt{1+y^2}}\,dy$

24) $\int_{-\infty}^\infty \frac{k}{1+k^2}\,dk$

25) $\int_{-\infty}^{\infty} \frac{c}{c^2 + 6 \cdot c + 25}\, dc$

26) $\int_{0}^{\pi} \frac{1}{a^2 + b^2}\, db$

27) $\int_{-\infty}^{\infty} \frac{1}{v^2 + 6 \cdot v + 10}\, dv$

28) $\int_{0}^{\infty} e^{-x}\, dx$

29) $\int_{0}^{\infty} x \cdot e^{-x^2}\, dx$

30) $\int_{1}^{\infty} \frac{1}{1 + x^2}\, dx$

31) $\int_{1}^{\infty} \frac{1}{x^2 \cdot \sqrt{x^2 - 1}}\, dx$

32) $\int_{1}^{\infty} \frac{1}{x^2 + x}\, dx$

33) $\int_{0}^{\infty} x^2 \cdot e^{-\frac{x^2}{2}}\, dx$

34) $\int_{2}^{\infty} \frac{1}{x^2}\, dx$

35) $\int_{0}^{\infty} x^2 \cdot e^{-x^3}\, dx$

36) $\int_{1}^{\infty} \frac{\ln(x)}{x^2}\, dx$

37) $\int_{1}^{e} \frac{1}{x \cdot \ln(x)}\, dx$

38) $\int_{1}^{\infty} \frac{1}{x \cdot \sqrt{1 + x^2}}\, dx$

39) $\int_{0}^{\infty} \frac{1}{\sqrt{(1 + x)^3}}\, dx$

40) $\int_{1}^{\infty} \frac{1}{x^2 + x^4}\, dx$

41) $\int_{0}^{1} x \cdot \ln(x)\, dx$

42) $\int_{0}^{\frac{1}{e}} \frac{1}{x \cdot \ln(x)^2}\, dx$

43) $\int_{1}^{e} \frac{1}{x \cdot \sqrt{\ln(x)}}\, dx$